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ABSTRACT

The recently reported parity (+) resonances are fitted as far as possible into the 225 physical states of the $\underline{4212}^{+}$-fold of $\widetilde{SU}(12)$. With these tentative assignments, none of the decay properties is in striking disagreement with the predictions of $\widetilde{SU}(12)$ invariance.

HIGHER MESON RESONANCES AND THE $\underline{4212}^{+}$ MULTIPLET OF $\widetilde{SU}(12)$

In its lowest meson multiplet of $\underline{35}$, the SU(6) symmetry scheme accommodates almost every one of the known 0^- and 1^- particles. The counterpart representation in $\widetilde{SU}(12)$ is $\underline{143}^-$ leaving room for an extra 0^- singlet, which can be identified as the $\bar{X}^0$ 960 MeV resonance. Now the next meson multiplet occurring in $\widetilde{SU}(12)$ in the $\underline{4212}^+$ and with the increasing number of recently reported parity (+) resonances [1] is our purpose in this note to attempt to fill up this representation and to determine the implications of $\widetilde{SU}(12)$ symmetry [2] on the various decay modes. A very similar representation to the $\underline{4212}^+$ is the $\underline{5940}^+$ but just for the sake of simplicity it will be neglected - as it is, there remain a host of gaps in the $\underline{4212}^+$ even after we have fitted in all of the fairly well-established particles. Thus the $\underline{4212}^+$ comprises, in addition to the $\underline{189}$ of SU(6) another $\underline{35} + \underline{1}$ states [3]. Its physical content and our assignments are given in the accompanying table.

Following the notation of Ref. 2 we briefly recapitulate for future comparison the results concerning the vertex function of three $\underline{143}^-$ multiplets. We let m and μ stand for the masses of the vector and scalar mesons [4]; then the $\widetilde{U}(12)$ invariant interaction is given by [5]

$$\mathcal{L} = \frac{g^2}{16\mu^2 m^2}\left[\begin{array}{l} i m^2 (m+2\mu) f^{ijk} (r-q)_\lambda \phi^i_\lambda(p)\phi^j_5(q)\phi^k_5(r) + \text{cyclic} \\ + 2\mu(m+2\mu) d^{ijk} \epsilon_{\lambda\mu\kappa\nu} p_\kappa q_\nu \phi^i_\lambda(p)\phi^j_\mu(q)\phi^k_5(r) + \text{cyclic} \\ + i\mu^2 f^{ijk}\phi^i_\lambda(p)\phi^j_\mu(q)\phi^k_\nu(r)\left\{\begin{array}{l} 3m\left[(q-r)_\lambda g_{\mu\nu} + (r-p)_\mu g_{\nu\lambda} + (p-q)_\nu g_{\lambda\mu}\right] + \\ + 4 q_\lambda r_\mu p_\nu / m \end{array}\right\} \end{array}\right] \quad (1)$$

0^+				1^+				2^+			
SU(3) Repn.	Particle	Mass (MeV)	Width (MeV)	SU(3) Repn.	Particle	Mass (MeV)	Width (MeV)	SU(3) Repn.	Particle	Mass (MeV)	Width (MeV)
1	ABC ?	315	15 ?	1	$\bar{X}_0$?	960	10 ?	1	K_1K_1 ?	1020	?
1	σ	400	80 ?	8	B Kρ $\bar{K}K^*$?	1215 1215 1410	120 60 60	8	f_0 A2 ? .	1250 1310 .	100 80 .
8	κ φ' .	725 520 .	10 ? 70 .	8	A1 ? . .	1090 . .	125 . .				
8	.	.	.	8	.	.	.				
27	.	.	.	10	.	.	.				
				10*	.	.	.				

Assignments of the parity (+) resonances within the $\underline{4212}^+$ multiplet of $\tilde{SU}(12)$. The data is taken from the tables of Rosenfeld et al. (Question marks denote less secure assignments or not well-established experimental values.)

and from the ρ decay width we deduce [6]

$$g_{\rho\pi\pi} = g\,(m+2\mu)/16\mu^2 \approx 2.8 \; .$$

The decomposition of the $\underline{4212}^+$ (represented by $\Phi^{[AB]}_{[CD]}$) has already been given [7]. The $\hat{SU}(12)$ invariant interaction

$$\mathcal{L} = g\,\bar{\Phi}^{[AB]}_{[CD]}(p)\;\Phi^{C}_{A}(q)\;\Phi^{D}_{B}(r) \tag{2}$$

reduces after some calculation to

$$\begin{aligned}
g^{-1}\mathcal{L} = & -(2\mu+M)\,\varphi^{[rs]}_{[pq]}(p)\,\phi_{5r}^{\;p}(q)\,\phi_{5s}^{\;q}(r)\;/12\mu \\
& + (M+m+\mu)\begin{bmatrix}(M+m-\mu)\,\epsilon_{\kappa\lambda\nu\mu}\,(\mu p_\mu - Mq_\mu) - \\ -\,2\epsilon_{\kappa\lambda\tau\mu}\,p_\mu q_\tau p_\nu\end{bmatrix}\frac{A_{\kappa\lambda}{}^{[rs]}_{[pq]}(p)\,\phi_{5r}^{\;p}(q)\,\phi_{\nu s}^{\;q}(r)}{4\mu m M^2} \\
& + (M+2m)^2\left[-g_{\mu\nu}M^2 + 2p_\mu p_\nu\right]\varphi^{[rs]}_{[pq]}(p)\,\phi_{\mu r}^{\;p}(q)\,\phi_{\nu s}^{\;q}(r)/24m^2M^2 \\
& + \begin{bmatrix}2M^2(M+2m)^2 g_{\mu\kappa}g_{\nu\lambda} - p_\mu p_\nu (q-r)_\kappa (q-r)_\lambda + \\ +\,4M(M+2m)\,g_{\nu\lambda}\,p_\mu\,(q-r)_\kappa\end{bmatrix}\frac{S_{\kappa\lambda}{}^{[rs]}_{[pq]}(p)\,\phi_{\mu r}^{\;p}(q)\,\phi_{\nu s}^{\;q}(r)}{8m^2M^2} \\
& + (M+2\mu)^2\,\eta^{\{rs\}}_{\{pq\}}(p)\,\phi_{5r}^{\;p}(q)\,\phi_{5s}^{\;q}(r)\;/16\mu^2 \\
& + (M+2m)^2\left[-g_{\mu\nu}M^2 + 2p_\mu p_\nu\right]\eta^{\{rs\}}_{\{pq\}}(p)\,\phi_{\mu r}^{\;p}(q)\,\phi_{\nu s}^{\;q}(r)/16m^2M^2 \\
& + \; .
\end{aligned}$$

(continued on next page)

$$+ \left[(M+\mu)^2 - m^2\right]\left[-g_{\mu\lambda}\{(M+m)^2-\mu^2\} + p_\mu (r-q)_\lambda\right] \frac{U_{\lambda[pq]}^{\{rs\}}(p)\, \phi_{\mu r}^{p}(q)\, \phi_{s\sigma}^{q}(r)}{5\sqrt{2}\,\mu m\, M^2}$$

$$+ (M+2m)\left[M\epsilon_{\lambda\nu\kappa\mu}(m p_\kappa - M r_\kappa) - 2\epsilon_{\lambda\nu\kappa\tau} p_\kappa r_\tau p_\nu\right] \frac{U_{\lambda[pq]}^{\{rs\}}(p)\, \phi_{\mu r}^{p}(q)\, \phi_{\nu s}^{q}(r)}{4\sqrt{2}\, m^2 M^2}$$

$$+ \text{Similar terms with } q \leftrightarrow r,\; U^{\{\}}_{[\,]} \rightarrow U^{[\,]}_{\{\}} .$$

(3)

We now turn to the decay properties predicted by (3):

<u>2+ Resonances</u>

$S\hat{U}(12)$ inhibits all but the interaction of the 2^+ with two 1^- mesons, a decay which is not physically realizable; consequently for the other available decay modes $2^+ \rightarrow 0^- + 0^-$, $2^+ \rightarrow 0^- + 1^-$ we must establish the <u>smallness</u> of the coupling constant (in relation to $g_{\rho\pi\pi}$ say) to indicate little symmetry breaking. The large decay widths (~ 100 MeV) would appear to belie this statement, but if due account is taken of the large phase space available and the <u>D -wave</u> nature of the decay into pseudoscalar mesons we find that $g_{f\pi\pi}$, $g_{f\omega\pi}$, g_{A_2KK}, $g_{A_2\pi\rho}$ are indeed small [8]. Thus from phenomenological couplings [9]

$$\mathcal{L}_{\binom{f\pi\pi}{AKK}} = g_{\binom{f\pi\pi}{AKK}} (q-r)_\kappa (q-r)_\lambda S_{\kappa\lambda}(p)\, \phi_5(q)\, \phi_5(r) / \mu$$

$$\mathcal{L}_{\binom{f\pi\omega}{A\pi\rho}} = g_{\binom{f\pi\omega}{A\pi\rho}} \epsilon_{\tau\mu\nu\kappa} (q-r)_\tau p_\mu \Phi_\nu(r)\, S_{\kappa\lambda}(p)\, (q-r)_\lambda \phi_5(q) / \mu M \qquad (4)$$

we obtain the decay widths

$$\Gamma_{\binom{f\pi\pi}{AKK}} = 3 g^2_{\binom{f\pi\pi}{AKK}} \left| q_{\binom{f\pi\pi}{AKK}} \right|^5 / 3\pi\mu^2 M^2_{\binom{f}{A}}$$

$$\Gamma_{\binom{f\omega\pi}{A\rho\pi}} = g^2_{\binom{f\omega\pi}{A\rho\pi}} \left| q_{\binom{f\omega\pi}{A\rho\pi}} \right|^5 / \pi\mu^2 M^2_{\binom{f}{A}}$$

(5)

to be compared with

$$\Gamma_{\rho\pi\pi} = g^2_{\rho\pi\pi} \left| q_{\rho\pi\pi} \right|^3 / 2\pi M^2_{\rho} \qquad (6)$$

Hence

$g_{\rho\pi\pi} : g_{f\pi\pi} : g_{f\omega\pi} : g_{AKK} : g_{A\rho\pi} \approx 1 : \frac{1}{20} : \frac{1}{15} : \frac{1}{20} : \frac{1}{5}$,

consistent with approximate $\tilde{SU}(12)$ invariance.

1^+ Resonances

Placing the B, $K^*\bar{K}$ and $K\rho$ resonances in a U-octet of eq. (3) (in the A-octet the decay $B \to \omega + \pi$ would be forbidden by isospin) we obtain the decay widths

$$\Gamma_{\binom{B\omega\pi}{K^*K}} = 3 g^2_{\binom{B\omega\pi}{K^*K}} \left| q_{\binom{B\omega\pi}{K^*K}} \right| / 8\pi M^2_{\binom{B}{K^*K}} \qquad (7)$$

From experimental values we get $g_{B\omega\pi} / g_{K^*K} \approx 0.75$ to be compared with the theoretical estimate of $\sqrt{6}/3 \approx 0.81$

No numerical statement is possible for the $K\rho$ resonance owing to the Q-value of -30 MeV! If we place the A_1 into the second U-octet, the decay width is the same as in (7) so the experimental data give $g_{A_1\rho\pi}/g_{B\omega\pi} \approx 0{\cdot}6$, which is not in violent disagreement with the theoretical prediction of unity [10].

<u>0^+ Resonances</u>

For simplicity we must assume that the singlet states are filled first; then if we place the ABC in the φ-singlet and σ in the η-singlet, $\widetilde{SU}(12)$ gives the theoretical ratios

$$g_{\sigma\pi\pi}/g_{ABC\pi\pi} = 3\,(M_\sigma + 2\mu)^2 / 8\sqrt{2}\,\mu\,(M_{ABC} + 2\mu) \approx 0{\cdot}9$$

$$g_{\sigma\pi\pi}/g_{B\omega\pi} = \frac{(M_\sigma + 2\mu)^2}{32\,\mu^2\sqrt{6}} \Big/ \frac{\{(M+\mu)^2 - m^2\}\{(M+m)^2 - \mu^2\}}{24\sqrt{2}\,\mu m M^2} \approx 0{\cdot}3$$

The experimental ratios of 1•6 and 0·7 respectively are at least of the same order of magnitude. As for the other SU(3) representation, it is conceivable that the ephemeral resonances σ' and κ fit partly into an octet, but in view of their rather dubious nature we will not attempt to extract any quantitative information about them.

In summary these results never show big disagreement with the demands of $\widetilde{SU}(12)$ invariance, and in the case of the better-established particles are sometimes numerically quite good. Nevertheless it would be premature to scrutinize or read too much into them in view of the tentative assignments of the few available (and still not experimentally well analyzed) resonances, mixing problems and parallel occurrence of <u>5940</u>$^+$ states only serving to muddy the issue. However the large multiplicity of these higher $\widetilde{SU}(12)$ meson resonances seem to call for more exhaustive experimental research.

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REFERENCES AND FOOTNOTES

(1) We base our work on the "Data on Elementary Particles and Resonant States" by A.H. Rosenfeld, A. Barbaro-Galtieri, W.H. Barkas, P.L. Bastien, J. Kirz and M. Roos. Rev. Mod. Phys., 977 (1964) 35.

(2) a. R. Delbourgo, A. Salam and J. Strathdee, Proc. Roy. Soc. 146 (1965) A 284.

b. R. Delbourgo, M.A. Rashid, A. Salam and J. Strathdee, Proc. Roy. Soc. (to be published)

c. A. Salam, J. Strathdee, J.C. Charap and P.T. Matthews, Phys. Letters (to be published)

d. See also B. Sakita and K.C. Wali, Phys. Rev. Letters (to be published)

(3) The reason why more physical states are accommodated is that $\widetilde{SU}(12)$ tracelessness does not imply SU(6) tracelessness. This question will be discussed more fully in a future paper.

(4) An ad hoc mass splitting between 0^- and 1^- states has been introduced since they remain disjoint even after application of the equations of motion. This was not done in Ref. 2. We make similar mass splittings later for the SU(3) multiplets within the $\underline{4212}^+$.

(5) As noted earlier (Ref. 2a) the $1^-0^-0^-$ and $1^-1^-1^-$ couplings are pure f whereas the $1^-0^-1^-$ coupling is pure d. Note also that $\varphi \to \pi\pi$ is forbidden by the Lipkin assignment of the physical φ. Equation (1) also gives

$$g_{\rho\omega\pi} / g_{\rho\pi\pi} = 2\mu / m^2 \approx 0{\cdot}44 / \mu$$

with mean masses of multiplets $m \approx 850$ MeV, $\mu \approx 400$ MeV.

This is in close agreement with the Gell-Mann, Sharp and Wagner estimate of $0{\cdot}47/\mu_\pi$.

(6) If the $\rho\pi\pi$ coupling is evaluated at <u>zero</u> momentum transfer it gets depressed by a factor $2\mu\,(m+2\mu)^{-1}$ down to about $1{\cdot}35$. (This can be seen from Eq. (6.6), Ref. 2a. There is actually an error in the expression for the $\pi\pi$ contribution to the vector current. In place of the factor $(1-q^2/\mu^2)$ one should read $(1+q^2/2\mu^2) \rightarrow (1+q^2/2\mu m)$ with our present mass splitting of 0^- and 1^- nonets.) In Ref. 2a we found $g_{\rho NN} = 3\mu\, g_{\pi NN}{}'5(\mu+2m_N) \approx$ $\approx 1{\cdot}3$ giving remarkably close agreement from the universality point of view.

(7) See especially footnote 6, Ref. 2b.

(8) As a general rule we use mean masses of multiplets when making a theoretical comparison of coupling constants. However when calculating coupling constants from the experimental decay widths we use <u>physical</u> masses to obtain a realistic appraisal of phase space effects.

(9) To define dimensionless coupling constants we use for our mass scale the pseudoscalar meson; this is a questionable point about our procedure.

(10) Placing the A_1 into an A-octet would lead to a serious discrepancy for the coupling constant ratios.

ERRATA

The second line of Eq. (1) should read

$2\mu\ (2m + \mu)$ in place of $2\mu\ (m + 2\mu)$

Delete Footnote (5) and replace by: (5). As noted earlier (ref. 2a) the $1^-0^-0^-$ and $1^-1^-1^-$ couplings are pure f whereas the $1^-0^-1^-$ coupling is pure d. Note also that $\varphi \to \rho\pi$ is forbidden by Lipkin's assignment of the physical φ. Eq. (1) also gives

$$g_{\rho\omega\pi} / g_{\rho\pi\pi} = 2\mu\ (2m + \mu) / m^2 (m + 2\mu) \approx 0.51 /$$

if we use mean masses of multiplets

$$m \approx 900 \text{ MeV} , \qquad \mu \approx 400 \text{ MeV}.$$

This is in close agreement with the Gell-Mann, Sharp and Wagner estimate of $0.47/\mu_\pi$ from ω decay. (An error in the Lagrangian of ref. 2b Eq. (3.1) has been corrected to derive this result; a factor of 2 should multiply the term $\phi^i_\kappa \phi^j_{\lambda\mu} \phi^k_{\nu\xi}$)