

Do Now: - Take a new hwk sheet from my desk

In your notes...

$$\lim_{x \rightarrow 4} \frac{x^2 - 16}{x - 4} = \frac{(x+4)(\cancel{x-4})}{\cancel{x-4}} \rightarrow \lim_{x \rightarrow 4} (x+4) = 4+4 = \boxed{8}$$

$\begin{matrix} 4-4 \\ = 0 \end{matrix}$

4/27/15

Aim: How do we find the limit of piecewise functions?

Limits of Piecewise Defined Functions

Ex. $f(x) = \begin{cases} 2x, & x < 0 \\ 5x, & x \geq 0 \end{cases}$

$-1, -2, -3, \dots \Rightarrow$
 $0, 1, 2, 3, \dots$

a. Find:

$$f(3) = 5(3) = 15$$

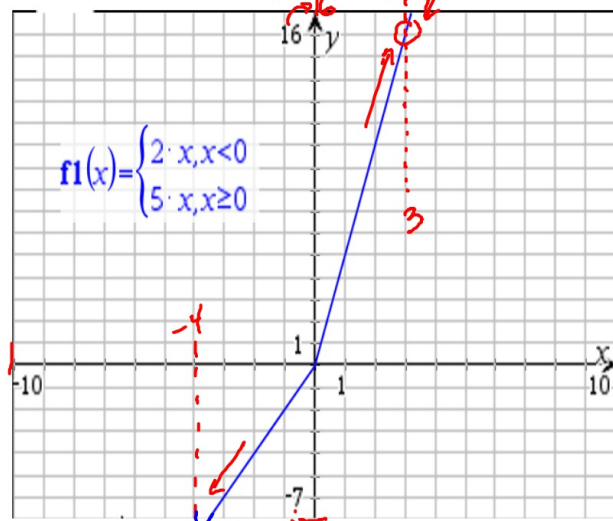
$$f(-3) = 2(-3) = -6$$

$$f(0) = 5(0) = 0$$

b. Find $\lim_{x \rightarrow 3} f(x) = 15$

use $5x$ ———→

Graphically

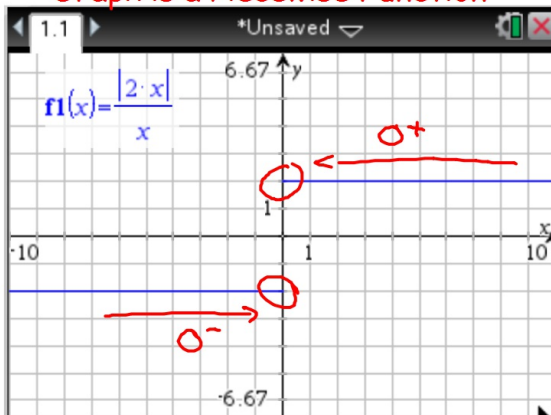


c. Find $\lim_{x \rightarrow -4} f(x) = 2(-4) = \boxed{-8}$

Recall One Sided Limits...

Ex.: (a) $\lim_{x \rightarrow 0} \frac{|2x|}{x}$

Graph is a Piecewise Function



(i) $\lim_{x \rightarrow 0^-} \frac{|2x|}{x} = -2$

(ii) $\lim_{x \rightarrow 0^+} \frac{|2x|}{x} = 2$

(iii) $\lim_{x \rightarrow 0} \frac{|2x|}{x} = \text{D.N.E.}$
b/c (i) $\neq$ (ii)

(b) $f(x) = \begin{cases} 4-x, & x < 1 \\ 4x-x^2, & x > 1 \end{cases}$. Find $\lim_{x \rightarrow 1} f(x) = \boxed{3}$

$\lim_{x \rightarrow 1^-} f(x) = \lim_{x \rightarrow 1} 4-x \rightarrow 4-1 = 3$ b/c $\lim_{x \rightarrow 1^-} = \lim_{x \rightarrow 1^+}$

$\lim_{x \rightarrow 1^+} f(x) = \lim_{x \rightarrow 1} 4x-x^2 \rightarrow 4(1)-1^2 = 4-1 = 3$

(c) Let $f(x) = \begin{cases} x-1, & x \leq 3 \\ 3x-7, & x > 3 \end{cases}$

(i) $\lim_{x \rightarrow 3^-} f(x) = x-1 = 3-1 = 2$

(ii) $\lim_{x \rightarrow 3^+} f(x) = 3x-7 = 3(3)-7 = 2$

(iii) $\lim_{x \rightarrow 3} f(x) = \boxed{2}$ b/c (i) = (ii)

(d) Let $g(t) = \begin{cases} t^2, & t \geq 0 \end{cases}$. Find:
 $t < 0$

(i) $\lim_{t \rightarrow 0^-} g(t) = t - 2 = 0 - 2 = -2$

(ii) $\lim_{t \rightarrow 0^+} g(t) = t^2 = 0^2 = 0$

(iii) $\lim_{t \rightarrow 0} g(t) = \text{D.N.E.}$ b/c (i) $\neq$ (ii)

(e) Find $\lim_{x \rightarrow 3} h(x)$ given that $h(x) = \begin{cases} x^2 - 2x + 1, & x \neq 3 \\ 7, & x = 3 \end{cases}$

$f(3) = 7$

$\lim_{x \rightarrow 3} h(x) \rightarrow \lim_{x \rightarrow 3} x^2 - 2x + 1 \rightarrow 3^2 - 2(3) + 1$
 $= 9 - 6 + 1$
 $= 4$