PUB-77-110

CURRENT ALGEBRA APPLIED TO PROPERTIES OF THE LIGHT HIGGS BOSON

William A. BARDEEN and S.-H.H. TYE
Fermi National Accelerator Laboratory, Batavia, IL 60510, USA

Received 26 January 1978

We present a general current algebra approach to the study of the new light Higgs boson h. In particular we apply it to the Weinberg–Salam model for both standard and non-standard Yukawa couplings. The partially conserved axial-vector current for his explicitly constructed and used to calculate its mass and lifetime.

The existence of pseudoparticle solutions [1] in non-abelian gauge theories has far-reaching consequences in the standard model for hadrons, namely, quantum chromodynamics (QCD). t' Hooft [2] and others [3] found that, in the proper vacuum, *P* and *CP* invariances are in general strongly violated. However, these violations are completely suppressed by the introduction of massless quarks. More recently, Peccei and Quinn [4] showed that these violations are suppressed even in the absence of massless quarks, provided at least one of the quarks obtains its entire mass via Yukawa coupling to a scalar field. The requirement for the restoration of *P* and *CP* invariance via this mechanism is the presence of a chiral U(1) symmetry in the original lagrangian. It is natural to identify this scalar field to be a Higgs scalar field in some unified theory of weak and electromagnetic interactions. It was pointed out by Weinberg and Wilczek [5] that, in this case, there will exist a very light neutral Higgs field. They further emphasized the possibility of its experimental discovery.

In this note we develop current algebra methods for computing properties of this Higgs particle which we shall call higglet (axion) [‡1]. We focus on estimates for the higglet mass and lifetime. Our methods will treat the strong interactions exactly and our predictions will depend only upon the structure of the weak interactions and the Higgs couplings.

We begin by studying the model originally proposed by Peccei and Quinn [4] although the results are easily generalized to more complex theories. Their theory involves an extension of the Weinberg–Salam model [6] which includes an additional Higgs doublet. One Higgs doublet couples only to 2/3 charged (p) quarks and the other only to −1/3 charged (n) quarks. This theory naturally conserves quark flavors in the Higgs sector and has an apparent additional U(1) symmetry which is broken by the QCD instanton effects. Peccei and Quinn [4] have shown that the instantons will not generate large *CP* violations in this class of theories due to the U(1) symmetry. The model is described by the lagrangian $\mathcal{L} = \mathcal{L}_{QCD} + \mathcal{L}_{WS} + \mathcal{L}_H$, where the first two terms are standard and $\mathcal{L}_H$ contains all the couplings involving the enlarged set of Higgs fields,

$$\chi_1 = \begin{pmatrix} \chi_1^0 \\ \chi_1^- \end{pmatrix}, \quad \chi_2 = \begin{pmatrix} \chi_2^+ \\ \chi_2^0 \end{pmatrix},$$

[‡1] This particle is called axion in ref. [5]. Following Peccei and Quinn, we call it "higglet"; we first heard this name from J.D. Bjorken.

$$\mathcal{L}_H = |(\partial_\mu + i\tfrac{1}{2}g\boldsymbol{\tau}\cdot\boldsymbol{A}_\mu + i\tfrac{1}{2}g'B_\mu)\chi_1|^2 + |(\partial_\mu + i\tfrac{1}{2}g\boldsymbol{\tau}\cdot\boldsymbol{A}_\mu - i\tfrac{1}{2}g'B_\mu)\chi_2|^2 - V(\chi_1,\chi_2)$$
$$\sum_{i,j}\Gamma^1_{ij}\{\bar{p}_{Ri}\chi_1^+\psi_{Lj} + \bar{\psi}_{Li}\chi_1 p_{Rj}\} - \sum_{i,j}\Gamma^2_{ij}\{\bar{n}_{Ri}\chi_2^+\psi_{Li} + \bar{\psi}_{Li}\chi_2 n_{Ri}\} - \sum_i \Gamma^\ell_i\{\bar{\ell}_{Ri}\chi_2^+\ell_{Li} + \bar{\ell}_{Li}\chi_2\ell_{Ri}\}, \quad (1)$$

where ψ_{Li} and ℓ_{Li} are the left-handed doublet fields for quarks and leptons respectively. Since we are interested in the properties of the higglet we focus on the Goldstone boson sector of the theory. The Higgs potential, V, is chosen such that both χ_1 and χ_2 have vacuum expectation values and such that there would be four Goldstone bosons in the absence of couplings to the gauge fields. Three of these particles are absorbed by the Higgs mechanism in generating mass for the $W^\pm_\mu$ and Z_μ mesons. The remaining Goldstone boson is identified as the higglet. Keeping only the higglet field, h, the Higgs fields χ_1 and χ_2 may be written as

$$\chi_1 = \frac{f_1}{\sqrt{2}}\begin{pmatrix}1\\0\end{pmatrix} e^{ixh/f}, \qquad \chi_2 = \frac{f_2}{\sqrt{2}}\begin{pmatrix}0\\1\end{pmatrix} e^{ih/xf}, \quad (2)$$

where $x = f_2/f_1$, $f = \sqrt{f_1^2 + f_2^2}$, and $\sqrt{2}G_W f^2 = 1$. The higglet lagrangian becomes

$$\mathcal{L}_h = m_W^2 W^+_\mu W^-_\mu + \tfrac{1}{2}m_Z^2 Z_\mu^2 + \tfrac{1}{2}(\partial_\mu h)^2 - \sum_i m_{p_i}\{\bar{p}_i p_i \cos(xh/f) + \bar{p}_i i\gamma_5 p_i \sin(xh/f)\}$$
$$- \sum_i m_{n_i}\{\bar{n}_i n_i \cos(h/xf) + \bar{n}_i\gamma_5 n_i \sin(h/xf)\} - \sum_i m_{\ell_i}\{\bar{\ell}_i\ell_i\cos(h/xf) + \bar{\ell}_i i\gamma_5\ell_i\sin(h/xf)\}, \quad (3)$$

where we have diagonalized the mass matrix. The residual U(1) symmetry is explicit in this lagrangian and is associated with the U(1) axial vector current defined by

$$J^s_{5\mu} = f\partial_\mu h + \frac{x}{2}\sum_i \bar{p}_i\gamma_\mu\gamma_5 p_i + \frac{1}{2x}\sum_i \bar{n}_i\gamma_\mu\gamma_5 n_i + \frac{1}{2x}\sum_i \bar{\ell}_i\gamma_\mu\gamma_5\ell_i. \quad (4)$$

This current is conserved except for the strong interaction anomaly. The anomalous divergence of this current depends on the total number of quark doublets, N, and is given by [7]

$$D^s = \partial_\mu J^s_{5\mu} = \tfrac{1}{2}(x + 1/x)N\{(g^2/16\pi^2)G^a_{\mu\nu}\tilde{G}^a_{\mu\nu}\}, \quad (5)$$

where $G^a_{\mu\nu}$ is the color gluon field strength tensor.

Although the divergence in eq. (5) is a total divergence, the existence of pseudoparticle effects [2,3] imply that the anomaly actually corresponds to a breakdown of symmetry in the appropriate vacuum [8]. Because of this symmetry breaking, we do not expect a physical massless Goldstone boson to be associated with the current of eq. (4). In this picture, the U(1) breaking which generates the η–π mass difference is much larger than the chiral symmetry breaking which generates the π mass. In the chiral symmetry limit, the higglet and the pion would become massless Goldstone bosons. The higglet is not to be identified with the current of eq. (4) but rather an anomaly-free current which becomes conserved in the chiral symmetry limit.

The two currents which are needed to study the chiral limit are the axial isospin current associated with the pion and the current to be associated with the higglet. They are defined as follows:

$$J^3_{5\mu} = \tfrac{1}{2}\bar{p}_0\gamma_\mu\gamma_5 p_0 - \tfrac{1}{2}\bar{n}_0\gamma_\mu\gamma_5 n_0, \quad J^h_{5\mu} = J^s_{5\mu} - \tfrac{1}{2}(x + 1/x)N\{(1+Z)^{-1}\bar{p}_0\gamma_\mu\gamma_5 p_0 + Z(1+Z)^{-1}\bar{n}_0\gamma_\mu\gamma_5 n_0\}, \quad (6)$$

where p_0 and n_0 are the usual light quarks and Z is the light quark mass ratio ($Z = m_p v_p/m_n v_n$). These currents are conserved in the limit $m_p, m_n \to 0$. Their divergences are given by:

$$D^3 = \partial^\mu J^3_{5\mu} = m_p\{\bar{p}_0 i\gamma_5 p_0\cos(xh/f) - \bar{p}_0 p_0\sin(xh/f)\} - m_n\{\bar{n}_0 i\gamma_5 n_0\cos(h/xf) - \bar{n}_0 n_0\sin(h/xf)\},$$

$$D^h = \partial^\mu J^h_{5\mu} = -(x + 1/x)N(1+Z)^{-1}m_p\{\bar{p}_0 i\gamma_5 p_0\cos(xh/f) - \bar{p}_0 p_0\sin(xh/f)\}$$
$$- (x + 1/x)NZ(1+Z)^{-1}m_n\{\bar{n}_0 i\gamma_5 n_0\cos(h/xf) - \bar{n}_0 n_0\sin(h/xf)\}. \quad (7)$$

The masses are determined from the vacuum expectation values of the sigma terms

$$\Sigma_0^{33} = \langle -\mathrm{i}[Q_5^3, D^3]\rangle_0 = -\{m_\mathrm{p} v_\mathrm{p} + m_\mathrm{n} v_\mathrm{n}\},\quad \Sigma_0^{3\mathrm{h}} = \langle -\mathrm{i}[Q_5^3, D^\mathrm{h}]\rangle_0 = 0,$$
$$\Sigma_0^{\mathrm{hh}} = \langle -\mathrm{i}[Q_5^\mathrm{h}, D^\mathrm{h}]\rangle_0 = -N^2(x+1/x)^2 Z(1+Z)^{-2}(m_\mathrm{p} v_\mathrm{p} + m_\mathrm{n} v_\mathrm{n}), \tag{8}$$

with

$$v_\mathrm{p} = \langle \bar{\mathrm{p}}_0 \mathrm{p}_0 \cos(x\mathrm{h}/f) + \bar{\mathrm{p}}_0 \mathrm{i}\gamma_5 \mathrm{p}_0 \sin(x\mathrm{h}/f)\rangle_0, \quad v_\mathrm{n} = \langle \bar{\mathrm{n}}_0 \mathrm{n}_0 \cos(\mathrm{h}/xf) + \bar{\mathrm{n}}_0 \mathrm{i}\gamma_5 \mathrm{n}_0 \sin(\mathrm{h}/xf)\rangle_0.$$

Using standard current algebra methods [9], these sigma terms may be used to compute the π mass, the higglet mass, and a mixing angle. The results are ($f_\pi \approx 94$ MeV)

$$f_\pi^2 m_\pi^2 = m_\mathrm{p} v_\mathrm{p} + m_\mathrm{n} v_\mathrm{n}, \qquad m_\mathrm{h}^2 = \sqrt{2}\, G_\mathrm{w} f_\pi^2 m_\pi^2 N^2 (x+1/x)^2 Z(1+Z)^{-2}, \tag{9}$$

and the value of the mixing angle is such that the higglet current defined in eq. (6) does not couple directly to the pion. We note that the higglet mass vanishes in the limit $m_\mathrm{p} \to 0$ ($Z \to 0$) or $m_\mathrm{n} \to 0$ ($Z \to \infty$) as expected from symmetry considerations. Heavy quarks will introduce corrections of order $(m_\mathrm{n} + m_\mathrm{p})/m_\mathrm{q}$.

The qualitative features of the results of eq. (9) may be understood in terms of an analogue theory where we replace the strong interaction theory by a U(2) nonlinear sigma model involving π mesons and an η meson which includes an explicit U(1) symmetry breaking interaction. The higglet mass may be computed directly with the result

$$m_\mathrm{h}^2 = (f_\pi^2/f^2) m_\pi^2 ((m_\eta^2 - m_\pi^2)/m_\eta^2) \sim (50\ \mathrm{keV})^2. \tag{10}$$

We note that the higglet mass vanishes in the limit of U(1) symmetry, $m_\eta^2 = m_\pi^2$, but in the physical situation, $m_\pi^2 \ll m_\eta^2$, the result becomes independent of the η mass and depends only on chiral breaking associated with m_π^2. We conclude that our calculation is insensitive to the details of the symmetry breaking due to the anomaly since we are close to the chiral symmetry limit. We expect a detailed treatment of the U(1) symmetry breaking will provide corrections of order m_π^2/m_η^2 to our results.

Since we have identified the correct current to be associated with the higglet, we can use current algebra to determine other properties of the higglet. The lifetime τ may be determined from the electromagnetic anomaly in the divergence of the higglet current in the same manner as the π^0 decay calculation.

From the form of the current defined in eq. (6), the h/π amplitude ratio is given by

$$fA(\mathrm{h} \to 2\gamma)/f_\pi A(\pi^0 \to 2\gamma) = \mathrm{Tr}(Q_\mathrm{h} Q^2)/\mathrm{Tr}(Q_3 Q^2) = N(x+1/x)Z(1+Z)^{-1}, \tag{11}$$

where Q is the electromagnetic charge matrix. From this we obtain the lifetime of h:

$$\tau(\mathrm{h} \to 2\gamma) = \tau(\pi^0 \to 2\gamma)(f^2/f_\pi^2)(m_\pi/m_\mathrm{h})^3 [\mathrm{Tr}(Q_\mathrm{h} Q^2)/\mathrm{Tr}(Q_3 Q^2)]^2, \tag{12}$$

or

$$\tau(\mathrm{h} \to 2\gamma) = \tau(\pi^0 \to 2\gamma)(m_\pi/m_\mathrm{h})^5 Z^{-1} = (0.4\ \mathrm{s}/Z)(100\ \mathrm{keV}/m_\mathrm{h})^5, \tag{13}$$

where the mass ratio $(m_\pi/m_\mathrm{h})^3$ in eq. (12) comes from phase space. The value of the quark mass ratio can be determined from electromagnetic mass differences of mesons [10] ‡2, $Z = m_\mathrm{p}/m_\mathrm{n} = 0.56$.

We will briefly consider theories which have Higgs couplings different than those of eq. (1). These theories will not have "natural" flavor conservation for the Higgs couplings but may be consistent with experiment if only the heavy quark couplings are modified (i.e. the two lightest quark doublets couple to the Higgs fields in the same way).

To be specific we consider a theory with N left-handed quark doublets, N left-handed lepton doublets, and two Higgs doublets. The necessary U(1) symmetry exists for theories where each right-handed fermion field couples to only one of the Higgs doublets. Current algebra estimates for the higglet mass and lifetime may be determined as

‡2 See also ref. [9]. A very small Z implies the (poor) mass relation $m^2(\mathrm{K}^0) - m^2(\mathrm{K}^+) \simeq 2m^2(\pi^0) - m^2(\pi^+)$. In fact, for $Z = 0$, the higglet is no longer needed to suppress P and CP violations in strong interactions.

before,

$$m_h^2 = m_\pi^2 \sqrt{2}\, G_w f_\pi^2 (x + 1/x)^2 (N_{p1} - N_{n1})^2 Z(1+Z)^{-2}, \tag{14}$$

$$\tau(h \to 2\gamma) = \tau(\pi^0 \to 2\gamma) \left(\frac{m_\pi}{m_h}\right)^5 \frac{(N_{p1} - N_{n1})^2 Z(1+Z)^{-2}}{((N_{p1} - N_{n1}) Z(1+Z)^{-1} + N_{n1} - N_{\ell 1})^2}, \tag{15}$$

where N_{p1} ($N_{n1}, N_{\ell 1}$) is the number of 2/3 quarks (−1/3 quarks, leptons) coupled to the first Higgs field, χ_1. We note that $N_{p1} - N_{n1}$ must be non-zero for the higglet mechanism to work as described by Peccei and Quinn [4]. In this case, N in the higglet current, eq. (6), is replaced by ($N_{p1} - N_{n1}$), where $J^s_{5\mu}$ is the appropriate canonical current.

We conclude that the current algebra methods may be directly applied to study properties of the proposed light Higgs boson since we are close to the light quark (p, n) chiral limit. These methods give unique predictions for the mass and lifetime of the higglet. Further current algebra predictions follow directly from the structure of the higglet current as determined by eq. (6).

After the completion of this work, we received a preprint [11] by S. Weinberg which contains a result similar to our mass formula, eq. (9). In our non-standard model, his result for higglet–pion mixing (eq. (3)) is modified to read

$$\xi_\pi = \frac{1}{2}\frac{f_\pi}{f}\left[(N_{p1} - N_{n1})\left(x + \frac{1}{x}\right)\frac{m_n - m_p}{m_n + m_p} - x + \frac{1}{x}\right],$$

while the h–η (SU(2) singlet) mixing is

$$C_{\eta h} = \tfrac{1}{2}(f_\pi/f)(N_{p1} - N_{n1} - 1)(x + 1/x).$$

For $N_{p1} = 2$ and $N_{n1} = 0$, these reduce to the case considered by him.

We would like to thank J.D. Bjorken, T. Goldman, T. Inami and R.D. Peccei for useful discussions.

References

[1] A.A. Belavin, A.M. Polyakov, A.S. Schwartz and Yu.S. Tyupkin, Phys. Lett. 59B (1975) 85.
[2] G. t' Hooft, Phys. Rev. Lett. 37 (1976) 8; Phys. Rev. D14 (1976) 3432.
[3] R. Jackiw and C. Rebbi, Phys. Rev. Lett. 37 (1976) 172;
C.G. Callan, R.F. Dashen and D.J. Gross, Phys. Lett. 63B (1976) 334.
[4] R.D. Peccei and H.R. Quinn, Phys. Rev. Lett. 38 (1977) 1440; Phys. Rev. D16 (1977) 1791.
[5] S. Weinberg and F. Wilczek, Talks presented at the B.W. Lee Memorial Conf. (1977) unpublished;
F. Wilczek, unpublished.
[6] S. Weinberg, Phys. Rev. Lett. 19 (1967) 1264;
A. Salam, in: Elementary particle physics, ed. N. Svartholm (Almquist and Wiksells, Stockholm, 1968) p. 367.
[7] S. Adler, Phys. Rev. 177 (1969) 2426;
J.S. Bell and R. Jackiw, Nuovo Cimento 60A (1969) 49;
W.A. Bardeen, Phys. Rev. 184 (1969) 1848.
[8] On the U(1) problem, see: J. Kogut and L. Susskind, Phys. Rev. D10 (1974) 3468; D11 (1975) 3594.
[9] M. Gell-Mann, R.J. Oakes and B. Renner, Phys. Rev. 175 (1968) 2195;
S.L. Glashow and S. Weinberg, Phys. Rev. Lett. 20 (1968) 224;
R. Dashen, Phys. Rev. 183 (1969) 1245.
[10] See, e.g.: S. Weinberg, Harvard Univ. preprint HUTP-77/A057.
[11] S. Weinberg, Harvard preprint HUTP-77/A074.